Local Bounded Cochain Projections and the Bubble Transform

Ragnar Winther, University of Oslo

The study of discretizations of Hodge Laplace problems in finite element exterior calculus unifies the theory of mixed finite element approximations of a number of problems in areas like electromagnetism and fluid flow. The key tool for the stability analysis of these discretizations is the construction of projection operators which commute with the exterior derivative and at the same time are bounded in the proper Sobolev norms. Such projections are referred to as *bounded cochain projections*. The canonical projections, constructed directly from the degrees of freedom, will commute with the exterior derivative, but unfortunately, they are not properly bounded. On the other hand, bounded cochain projections have been constructed by combining a smoothing operator and the unbounded canonical projection. However, an undesired property of these *smoothed projections* is that, in contrast to the canonical projections, they are nonlocal. Therefore, we have recently proposed an alternative construction of bounded cochain projections, which also is local. This construction can be seen as a variant of the well known Clément operator, and it utilizes a double complex structure defined on the macroelements associated the subsimplexes of the grid. In addition, we will also discuss a new tool for analysis of finite element element methods, referred to as *the bubble transform*. In contrast to all the projections operators above, this transform will lead to projections with bounds which are independent of the polynomial degree of the finite element spaces. As a consequence, this can potentially simplify the analysis of the so-called p-method.